

No of pages: 1

Time allowed: 45 min

Total marks: 15

Question 1 (8 Marks)

(a) True or false:

- The mapping, $z = e^{Ts}$, from continuous to discrete-time system is valid for both poles and zeros.
- The transfer function of a system is the same as the z-transform of the impulse response of the system.
- With the mapping, $z = e^{Ts}$, the unit circle in s-plane is mapped to the imaginary axis in z-plane.
- The pulse response of a first order system having a pole at $z = 0.5$ will have an alternating sign.
- With trapezoidal approximation, continuous-time stable filters are always converted to stable discrete-time filters.
- Zero-order hold is a non-causal operation.

(b) Consider a signal of frequency 1 Hz, sampled at a rate of $f_s = 10$ Hz. Assume that there is a noise component at 8 Hz.

- 1- What are the first four frequencies appearing at the sampled signal?
- 2- How to eliminate the effect of aliasing?

Question 2 (7 Marks)

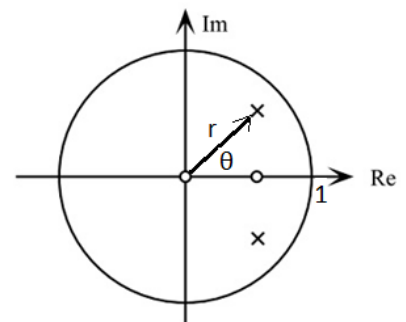
(a) Given the following signal in the z-domain

$$Y(z) = \frac{(z+1)}{(z-1)(z-2)}$$

- 1- What is the final value of $y(k)$?
- 2- Find the inverse z-transform of $Y(z)$.

(b) A system has two poles, two zeros and a dc gain of 10. The location of the poles and zeros are shown in the given figure, $r = 1/\sqrt{2}$, $\theta = 45^\circ$.

- 1- Find the transfer function of the system.
- 2- Find the first four samples of the unit pulse response of the system.



----- All best wishes -----

Answer

Question 1

- (a) $X \sqrt{X} X \sqrt{X}$
(b) 1, 2, 8, 9 Hz – use an anti-aliasing filter of cut-off frequency 5 Hz.

Question 2

- (a) 1- As $Y(z)$ has a pole at $z = 2$ (outside the unit circle), the final value of $y(k)$ has a final value of ∞ .

2-

$$\frac{Y(z)}{z} = \frac{(z+1)}{z(z-1)(z-2)} = \frac{A}{z} + \frac{B}{(z-1)} + \frac{C}{(z-2)} = \frac{0.5}{z} - \frac{2}{(z-1)} + \frac{1.5}{(z-2)}$$
$$Y(z) = 0.5 - \frac{2z}{(z-1)} + \frac{1.5z}{(z-2)}$$
$$y(k) = 0.5\delta(k) - 2 + 1.5(2)^k$$

Again, we can see that the final value of $y(k)$ is ∞ .

- (b)

1- The zeros are $z = 0, z = \frac{1}{\sqrt{2}} \cos 45^\circ = \frac{1}{2}$

The poles are $z = \frac{1}{\sqrt{2}} (\cos 45^\circ \pm j \sin 45^\circ) = \frac{1}{2} \pm j \frac{1}{2}$

Hence the transfer function is

$$G(z) = \frac{z(z-0.5)}{(z-0.5+j0.5)(z-0.5-j0.5)} = \frac{z(z-0.5)}{z^2 - z + 0.5}$$

The dc gain of $G(z) = G(1) = 1$. For the transfer function to have a dc gain of 10, the transfer function is:

$$G(z) = 10 \frac{z(z-0.5)}{z^2 - z + 0.5}$$

- 2- Using long division, the pulse response is given by:

$$Y(z) = 10 + 5z^{-1} + 0z^{-2} - 2.5z^{-3} + \dots$$

That is, $y(0) = 10, y(1) = 5, y(2) = 0, y(3) = -2.5$.